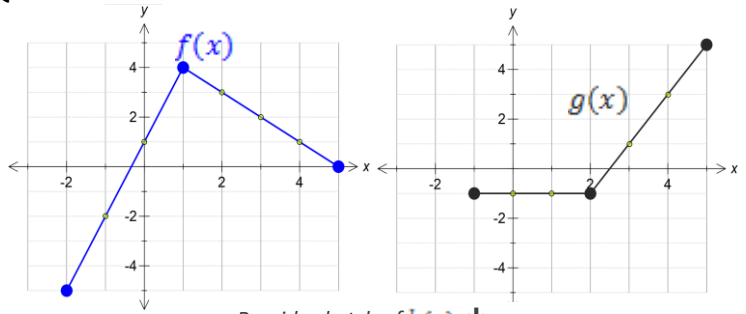


VM30-1 Function Operations & Polynomials Quiz

NO CALCULATORS are allowed for this quiz!



1. Given the graphs of  $f(x)$  and  $g(x)$  on the right...

(a) State the domain of  $h(x) = f(x) - g(x)$

1  $-1 \leq x \leq 5$   
 or ...  $[-1, 5]$

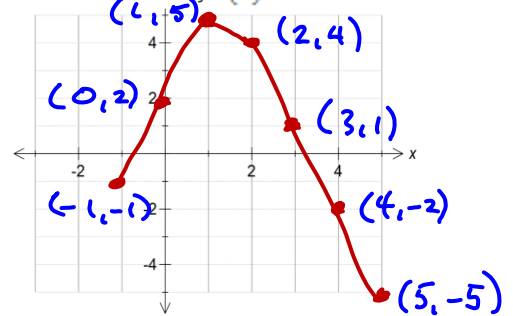
(b) State the indicated values in the blanks provided

3 i)  $f(3)$  2  
 ii)  $f \circ g(0)$  -2  $g(0) = -1, f(-1) = -2$   
 iii)  $f \circ f \circ g(0)$  -5  $f \circ g(0) = -2, f(-2) = -5$

(c) Provide a sketch of  $h(x) = (f - g)(x)$  on the grid provided. →

1

Provide sketch of  $h(x)$ .



2. Two functions are defined by  $f(x) = x^2 - 1$  and  $g(x) = 2x + 3$ .

(a) State the expression (fully simplified) for a function  $(f + g)(x)$  and state the domain.

2  $\underbrace{x^2 - 1}_f + \underbrace{2x + 3}_g = \underline{x^2 + 2x + 2}$  Simplified  $(f + g)(x)$   
 DOMAIN of  $(f + g)(x)$   $\mathbb{R}$

(b) State the expression (do not simplify) for a function  $(\frac{f}{g})(x)$  and state the domain.

5 2 Domain:  $2x + 3 \neq 0$   
 $\frac{2x}{2} \neq \frac{-3}{2}$   
 $\underline{\frac{x^2 - 1}{2x + 3}}$  Non-simplified  $(\frac{f}{g})(x)$   
 DOMAIN of  $(\frac{f}{g})(x)$   $x \neq -\frac{3}{2}$

(c) State the expression (simplified) for a function  $f \circ g \circ g(x)$ .

1  $f(g(g(x)))$   
 $f(g(2x + 3))$   
 $f(2(2x + 3) + 3)$   
 $(2(2x + 3) + 3)^2 - 1$   
 $(4x + 9)^2 - 1$   
 $16x^2 + 72x + 81 - 1$   
 $\underline{16x^2 + 72x + 80}$  Simplified  $f \circ g \circ g(x)$

3. Two functions are defined by  $f(x) = \sqrt{x - 1}$  and  $g(x) = x^2$ . Determine the expression for a function  $f \circ g(x)$ , and state the domain.

2  $f(x^2) = \underline{\sqrt{x^2 - 1}}$   
 $\underline{\sqrt{x^2 - 1}}$   $f \circ g(x)$   
 DOMAIN of  $f \circ g(x)$   $(-\infty, -1] \cup [1, \infty)$   
 $x \leq -1$  or  $x \geq 1$

4. Two functions are defined by  $f(x) = \sqrt{x - 1}$  and  $g(x) = x^2 - x$ . Determine the value of  $g \circ f(10)$ .

1  $f(10) = \sqrt{10 - 1}$   
 $= \sqrt{9}$   
 $= 3$   
 So,  $g(3) = (3)^2 - (3)$   
 $= 9 - 3$   
 $= 6$   
 Value of  $g \circ f(10)$  6

5. Below is a list of functions. Indicate each polynomial functions but placing a "P" beside it. Then, indicate the degree of the polynomial function.

$$f(x) = 3x^2 + 5x - 7$$

P Degree 2

$$y = 5x(x - 2)(x + 3)$$

P Degree 3

$$y = 3\sqrt{x-1} + 2$$

~~X~~

$$y = \sqrt{5x} - 7$$

P Degree 1

$$y = 3x^2 + 2x^{-1}$$

~~X~~

$$y = 6x^2(x + 5)^7$$

P Degree 9

3

5. For each provided equation on the left, match with the appropriate numbered graph. **Note:** There are more graphs than equations – some graphs are "extras"!

a)  $f(x) = 3x^2 + 5x - 7$

4

b)  $2y = 5x + 8$

$y = 2.5x + 4$

6

c)  $y = x^3 - 4x^2 + x + 6$

1

d)  $y = -(x + 1)(x + 2)(x - 3)$

9

e)  $f(x) = x^4 - 17x^2 + 16$

2

f)  $f(x) = -2(x + 1)(x - 1)(x - 3)(x - 4)^2$

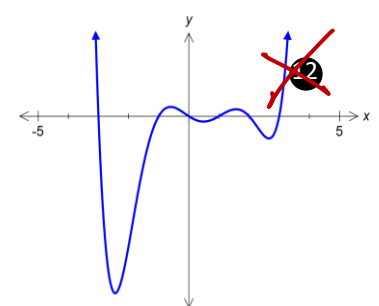
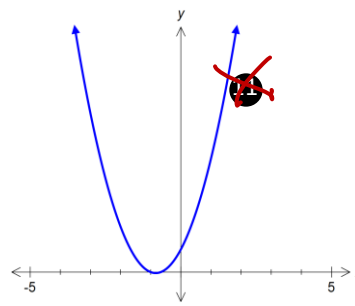
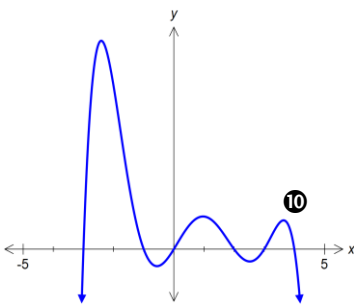
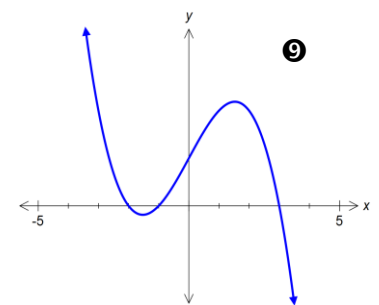
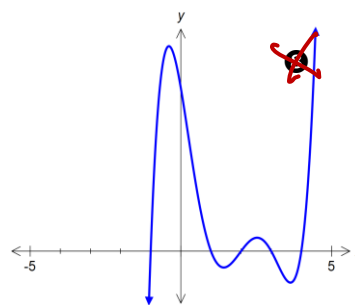
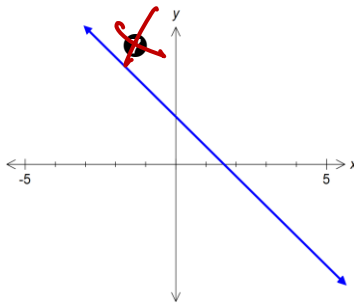
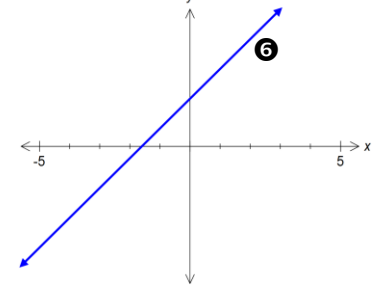
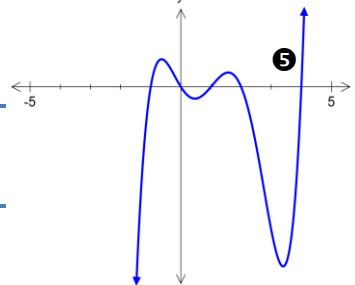
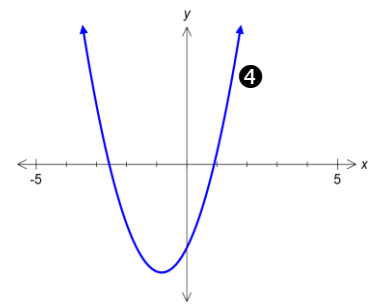
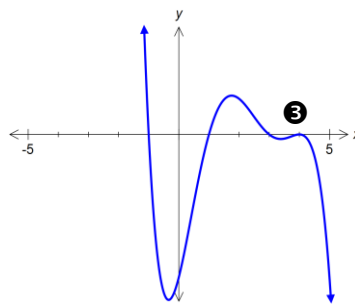
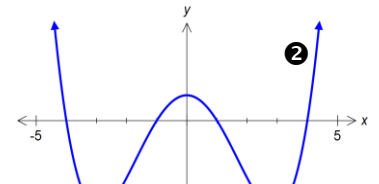
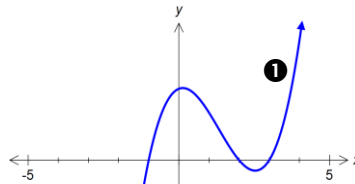
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g)  $y = -x^6 + 5x^5 + 7x^4 - 53x^3 + 18x^2 + 72x$

10

h)  $y = 2x^5 - 12x^4 + 14x^3 + 12x^2 - 16x$

5



8

1. Determine the value of  $k$  such that when  $f(x) = x^4 + kx^3 - 3x - 5$  is divided by  $x - 3$ , the remainder is

$$-14 \rightarrow P(3) = 14$$

$$(3)^4 + k(3)^3 - 3(3) - 5 = -14$$

$$81 + 27k - 9 - 5 = -14$$

$$27k = -81$$

$$k = -3$$

2. Use the factor / remainder theorem to show that:

(a)  $(x + 3)$  is a factor of  $P(x) = x^3 - 7x + 6$

$$\text{Show } P(-3) = 0$$

(division gives no remainder)

$$P(-3) = (-3)^3 - 7(-3) + 6$$

$$= -27 + 21 + 6$$

$$= 0 \quad \checkmark \quad \text{Factor}$$

(b)  $(x - 1)$  is NOT a factor of  $f(x) = 2x^3 + 9x^2 + 7x - 6$

$$\text{Show } P(1) \neq 0$$

$$2(1)^3 + 9(1)^2 + 7(1) - 6$$

$$= 2 + 9 + 7 - 6$$

$$= 12 \quad \leftarrow \text{that's not zero!}$$

NOT a Factor

3. Use synthetic division to find the result when  $P(x) = x^3 - 3x^2 - 4x + 12$  is divided by  $x - 2$ .

$$\begin{array}{r|rrrr}
 2 & 1 & -3 & -4 & 12 \\
 & \downarrow & 2 & -2 & -12 \\
 \hline
 & 1 & -1 & -6 & 0
 \end{array}$$

Remainder

$$\text{Result: } \underline{\underline{x^2 - x - 6}}$$

4. FULLY FACTOR  $P(x) = x^3 - 4x^2 + x + 6$  using an algebraic method. Show all steps / reasoning.

Potential zeros:  $\pm 1, \pm 2, \pm 3, \pm 6$

$$P(1) = (1)^3 - 4(1)^2 + (1) + 6 \neq 0$$

$$P(-1) = (-1)^3 - 4(-1)^2 + (-1) + 6 = 0 \checkmark \text{ Factor}$$

So,  $(x+1)$  is a factor!

Find remaining factors by dividing  
 $(x^3 - 4x^2 + x + 6) \div (x+1)$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \leftarrow R \end{array}$$

$x^2 - 5x + 6$

$$\text{So, } P(x) = (x^2 - 5x + 6)(x+1)$$

$$P(x) = (x-3)(x-2)(x+1)$$